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THE EFFECT OF SOLID ADMIXTURES ON THE VELOCITY

OF MOTION OF A FREE DUSTY AIR JET

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In dusty air flows occurring in industrial practice in transport by air pressure of friable materials, in the drying, annealing, and so forth, of a pulverized solid mass in suspension, and in other processes, the concentration of solid particles usually has a magnitude of the order of 1 kg per 1 kg of air. At such a concentration, the ratio of the volume of the particles to the volume of the air is small (less than one-thousandth part). However, regardless of this, the presence of a solid admixture manifests itself in the rules for the velocity distribution of the air in a dusty air flow. As a result, the rules of velocity change are different for clean and for dusty air flows. The estimation of the influence of the admixture on the velocity of the motion of the flow presents a definitive interest. One of the attempts to estimate that influence on the axial velocity of a free axially symmetrical jet with admixtures was made by Abramovich (ref. 1).

Abramovich assumed beforehand that the fine particles of the admixture in the jet are subject to the motion of the air (that is, that the velocity of the admixture is approximately equal to the local velocity of the air); he then took as the basis of his considerations, in solving the problem, the condition that the amount of motion of the two-phase jet must be constant.

Proceeding from these assumptions, Abramovich obtained the relationship of the velocity of the air moving along the axis of the axially symmetrical jet in relation to the distance to the origin of the jet in the following form

$$\frac{u_m}{u_0} = \frac{0.96}{\frac{as}{R_0}} \sqrt{\frac{1 + C_0 \frac{w_0}{u_0}}{1 + \frac{0.567}{\frac{as}{R_0}} C_0}} \tag{1}$$

where u_m and u_0 are, respectively, the velocity of the air on the axis and in the initial section of the jet; s the distance from the origin along the axis of the jet; R_0, the initial radius of the jet; w_0, the initial velocity of the admixture; C_0, the initial concentration of the particles; and a, the coefficient of turbulence of the jet.

As can be seen, the expression (1) does not take into account the influence of the relative velocity of the particles of the admixture in the volume of the jet; the term $\dfrac{w_0}{u_0}$ enters into it which takes into account only the influence of the initial relative velocity.

In the reports of several authors (refs. 3 and 4) it was shown that in dusty air flows, particularly in free flows, as a result of the large difference in the masses of the solid admixture and of the air, the particles of the admixture move with a velocity which is different from the velocity of the air, that is, there exists in the jet an appreciable relative velocity of the air and of the particles even when their dimensions are comparatively small (60 μ - 400 μ). Therefore, it is logical to estimate the influence on the velocity of the jet not only of the initial relative velocity but also of the relative velocity in the entire volume of the jet.

Such an attempt is described in the present note.

In reference 4 it was shown that the ratio of the velocity of the particles to the velocity of the air varies only slightly with respect to the cross section of the jet. It refers particularly to the basic portion. Only in the cross sections of the initial portion does this ratio undergo more or less appreciable changes. This experimental fact permits estimating, in first approximation, the ratio of the velocity of the dust particles to the velocity of the air as a constant through the cross section. Such an assumption makes it possible to introduce into the formula of Abramovich (1) a correction for the inequality of the values of the velocities of the particles and of the air in the jet.

On the basis of the calculation, the condition of constancy of the momentum of the two-phase jet was assumed (just as was done by Abramovich).

Through the element of area of a circular free jet $dF = 2\pi y\,dy$ of any arbitrary cross section there passes, during the time $d\tau$, an amount of gas $dG_a = 2\pi y\,dy\rho u\,d\tau$. With this gas there passes through the same element of area an amount of dust $dG_d = 2\pi\rho cy\,dy u\,d\tau$. Here the following symbols are introduced, in addition to the accepted ones: $c = \dfrac{dG_d}{dG_a}$ - mass concentration of dust, ρ - air density, τ - time, y - distance from the axis in radial direction. The total flow of momentum through the whole selected cross section under the condition of constancy of the amount of motion of the dusty air jet is written in the following form

$$\int_0^\infty 2\pi\rho u(u + cw)y\,dy = \frac{G_{a.o}}{g}u_0 + \frac{G_{d.o}}{g}w_0 \qquad (2)$$

where $G_{d.o}$ and $G_{a.o}$ are, respectively, the discharges of dust and of air through the initial section of the jet, g - the acceleration of gravity.

The condition of constancy of the ratio of the velocities of the dust and of the air with respect to the cross section permits rewriting the right-hand side of equation (2) in the form

$$\int_0^\infty 2\pi\rho u^2 y\,dy + k \int c2\pi\rho u^2 y\,dy \qquad (A)$$

where $k = \dfrac{w}{u}$.

Having denoted also $k_0 = \dfrac{w_0}{u_0}$, $c_0 = \dfrac{G_{d.o}}{G_{a.o}}$ and going over to the new variable $\varphi = \dfrac{y}{as}$, we transform equation (2) in the following manner

$$2\pi(as)^2\rho u_m^2 \int_0^{\varphi_{rp.}}\left(\frac{u}{u_m}\right)^2\varphi\,d\varphi + 2\pi(as)^2\rho u_m^2 k \int_0^{\varphi_{rp.}}\left(\frac{u}{u_m}\right)^2 c\varphi\,d\varphi =$$

$$\pi R_0^2 \rho_0 u_0^2\left(Hc_0 k_0\right) \qquad (3)$$

or

$$\left(\frac{u_m}{u_0}\right)^2 = \frac{1 + c_0 k_0}{2\left(\dfrac{as}{R_0}\right)^2\left[\displaystyle\int_0^{\varphi_{rp.}}\left(\frac{u}{u_m}\right)^2\varphi\,d\varphi + k \int_0^{\varphi_{rp.}}\left(\frac{u}{u_m}\right)^2 c\varphi\,d\varphi\right]} \qquad (4)$$

The values of the integrals occurring in the denominator of the right-hand side are known from the theory of jets (ref. 1).

Having assumed the relation between the changes of concentration and of velocity in the jet cross section to be the following: $\dfrac{c}{c_m}\sqrt{\dfrac{u}{u_m}}$, we substitute the values of the integrals in equation (4) as

$$\left(\frac{u_m}{u_0}\right)^2 = \frac{1 + c_0 k_0}{2\left(\dfrac{as}{R_0}\right)^2\left(0.535 + k\,\dfrac{0.303c_0}{\dfrac{as}{R_0}}\right)} \qquad (5)$$

From the latter ratio we obtain the rule for the variation of the axial velocity of the two-phase flow with the correction k for the inequality of the velocities of the dust particles and of the air

$$\frac{u_m}{u_0} = \frac{0.96}{\frac{as}{R_0}} \sqrt{\frac{1 + c_0 k_0}{1 + \frac{0.567}{\frac{as}{R_0}} c_0 k}} \tag{6}$$

As can be seen from the ratio (eq. 6), the influence of the difference of the velocities of the dust particles and of the air, taken into account by the coefficient k, is analogous to the influence of the variation of the initial concentration of the admixture in formula (1) since the quantities c_0 and k enter into equation (6) in the form of a product.

It is readily seen that in the case of absence of admixtures in the jet $\left(c_0 = 0\right)$, equation (6), like equation (1), also goes over into the well-known law of variation of the axial velocity of the free single-phase jet (ref. 1).

If we assume the velocities of the dust particles and of the air in the flow to be equal ($k = 1$), equation (6) leads to Abramovich's formula (1).

Let us now consider the analysis of the influence of the quantity k on the velocity of the air in the jet.

According to experiments (ref. 4), the maximum value of the ratio k in the jet attains the magnitude 2 (at an initial velocity of the jet of the order of 30 m/sec). On the average, this quantity has a smaller value (about 1.5). Therefore, as shown by the calculation according to formula (6), the influence of the dust particles on the motion of the air, under the conditions of reference (4), is in general insignificant.

The calculated curves $\frac{u_m}{u_0} = f\left(\frac{as}{R_0}\right) c$ with a correction for the quantity $c_0 k$ are presented in figure 1. The parameter for these curves is the product $c_0 k$ $\left(\text{in this manner there corresponds to the value } c_0 k = 2, \text{ for instance: } c_0 = 1 \text{ and } k = 2, \text{ or } c_0 = 2 \text{ for } k = 1, \text{ etc}\right)$.

It can be seen from the figure that the curves 2 and 3, corresponding to the ratios $k = 1.5$ and 2 for $c_0 = 1$ lie only slightly above the curve 1 which corresponds to $c_0 = 1$ and $k = 1$ (that is, in the absence of a relative velocity).

The quantity k exerts an appreciable influence only for values higher than 2 (for instance, for $k = 5$, the velocity of the air increases by 30 to 50 percent in comparison with $k = 1$).

The absence of an appreciable influence of the particles on the aerodynamics of the jet (for a comparatively small velocity of the jet and for concentrations of dust not larger than 0.5 to 1.0 kg/kg) was noted also by Kubynin (ref. 2) in his investigation of a free dusty jet.

Summing up our findings, it must be noted that for a mass concentration of particles in an air (gas) jet of less than 1.0 kg/kg and an initial jet velocity of less than 30 m/sec (ref. 4), the presence of dust particles has practically no effect on the aerodynamics of the air jet.

The velocity of the air (gas) in that case can be calculated just as for a single-phase flow. In the case where the concentration or the quantity k have higher values than those indicated, the calculation necessarily leads to the function (6).

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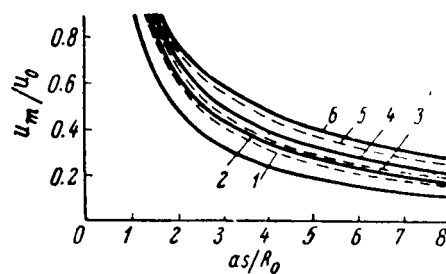   Kazakhstan Socialist Republic), No. 8, 1955.

Figure 1.- Variation of the velocity of the air along the axis of the two-phase jet. (The values $c_0 k = 1.5$, 2, 3, 5, 7, 5 correspond to the numbers of the curves 1 to 6. The solid curve not denoted by a number signifies a jet without admixtures.)